Effect of a Wing on Its Tip Vortex

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A theoretical and an experimental analysis was performed to determine the effect that the wing geometry and the boundary layer on the lower surface of the wing have on the trailing rolled-up tip vortex. Model test results are presented for the vortex structure downstream of wings of varying aspect ratio and taper ratio. The geometry and strength of the rolled-up tip vortices were measured over a range of lift coefficients. In general, it was found that the strength of the vortex is less than one would predict on the basis of the midspan circulation about the wing. Conclusions are given with regard to the size and strength of a vortex as they relate to the wing aspect ratio, taper ratio, and lift coefficient.

Nomenclature

AR = aspect ratio
c = wing chord
c_0 = midspan chord length
c_l = section lift coefficient
C_L = wing-lift coefficient (total lift/$\frac{1}{2}\rho VS$)
k = vorticity meter calibration constant
n = exponential constant
r = radial coordinate
R = tip radius of the vanes
S = wing planform area
t = time
V = freestream velocity
V_θ = tangential velocity
V_x = axial velocity
w = vortex width
α = absolute angle of attack
ζ = vorticity
ζ_0 = maximum vorticity
Γ = strength of circulation
Γ_0 = midspan value of bound circulation
η = vorticity meter calibration factor
λ = taper ratio
ν = kinematic viscosity
ρ = freestream density

Introduction

A LIFTING wing moving through a fluid induces a differential pressure field between the upper and lower surfaces. The resultant spanwise movement of fluid combined with the freestream velocity generates a helical movement of fluid about the wing tip and a vortex sheet behind the wing. This vortex sheet is unstable and rolls up into two distinct vortices within a few chord lengths downstream of the trailing edge. The structure of such a vortex is shown in Fig. 1. The vortex is characterized by two distinct regions of flow. Within a region called the core the flow is predominantly rotational and the tangential velocity varies nearly linearly with the radius, going from zero at the centerline of the vortex, to a maximum at the edge of the core. Outside this region the flow is nearly irrotational, and the tangential velocity varies inversely with the radius, asymptotically approaching a value of zero. The corresponding vorticity distribution for the vortex has a maximum value at the centerline and decreases asymptotically to zero. The area integral of the vorticity taken over a transverse plane within the rolled-up vortex should equal the maximum value of the bound circulation around the wing.

This investigation considers the aspect ratio, taper ratio, and the extent of turbulent flow on the lower surface of the wing as parameters that affect the flow over a wing surface and could influence the size and strength of the rolled-up tip vortex. The experimental investigation utilized a small vorticity meter, or vortex probe, a fluorescent oil film flow-visualization study, and a five-holed pressure probe. The vorticity meter measured the vorticity distribution within the rolled-up vortex with a set of small, unpitched vanes mounted on jewel bearings so as to rotate approximately at the same rotational speed as the fluid at a point. To investigate the manner in which the boundary layer on the lower surface of the wing influenced the tip vortex, a flow-visualization study was conducted. The five-holed pressure probe was used to evaluate the calibration of the vortex probe and the accuracy of the vorticity data. From the experimental results, a prediction of the nature of the trailing vortex can be made for any wing of comparable geometry and Reynolds number.

Calibration of the vorticity meter was necessary to improve the accuracy of the data. Previously, this calibration was assumed to vary with the freestream velocity, which in turn was considered constant throughout the flowfield.[1,2] However, a recent study[3] found the axial velocity to vary across the vortex and suggested that the calibration be so adjusted.

Theoretical Investigation and Discusssion

Theoretically, the total circulation within the rolled-up tip vortex equals the midspan value of the bound circulation around the finite wing. Through this relationship the char-

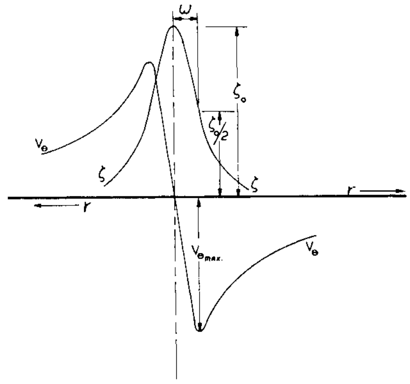

Fig. 1 Structure of the vortex.

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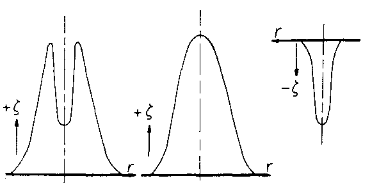

Fig. 2 Method of superposition.

acteristics of the wing can be associated with those of the vortex.

The vorticity is equal to the curl of the velocity vector. The vorticity in polar coordinates can be integrated to obtain the tangential velocity as a function of the radius R,

$$V_\theta = \frac{1}{R}\int_0^R \zeta r dr \tag{1}$$

Next consider the Navier-Stokes equations of motion for axisymmetric flow in polar coordinates. If it is assumed that there exists initially a potential vortex of strength Γ_0, then one possible solution to the equation is

$$\zeta = (\Gamma_0/4\pi\nu t)\, e^{-(r^2/4\nu t)} \tag{2}$$

Combining Eqs. (2) and (1) tangential velocity becomes

$$V_\theta = (\Gamma_0/2\pi r)[1 - e^{-(r^2/4\nu t)}] \tag{3}$$

The foregoing was first obtained by Lamb[4] and defines the radial distribution of the velocity through the vortex as a function of time. Thus, a relationship between the properties of the vortex is available.

To fit the vorticity data obtained from the experimental investigation, Eq. (2) was used in the form

$$\zeta = \zeta_0\, e^{-(r/w)^n \log_e 2} \tag{4}$$

ζ_0 is the maximum vorticity and w is defined as the vortex width and is equal to the radius for which the vorticity is one-half its maximum value (see Fig. 1). Therefore, given ζ_0 and the values of ζ for two values of r, the constants w and n were obtained. For a proper choice of the value of n in Eq. (4), the expression could be made to fit all of the experimental data, with the exception of that taken in a region of vortex "breakdown." The values of w and n are substituted into Eqs. (1) and (4), which are numerically integrated by means of a digital computer to obtain the circulatory strength and the tangential velocity as a function of the radius.

For the data obtained in a region of vortex breakdown a method of superposition was adopted. The proposed technique assumed that the vorticity distribution obtained in a region of vortex breakdown could be represented by the superposition of two normal vorticity distributions, one of which is negative, as shown in Fig. 2. The distributions are so constructed that the sum of the vorticity of the two distributions taken at any radial distance is equal to that of the original distribution at the same distance. Thus, it can be shown that the sum of the circulation computed for the two proposed distributions is equal to the circulation of the original vorticity distribution.

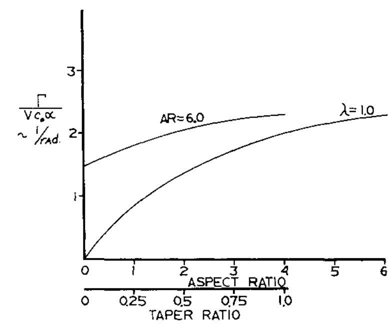

Fig. 3 Theoretical variation of the circulation.

Table 1 Test wing characteristics

Wing	AR	λ	C_{l_0}/C_L	$C_{L\alpha_{exp}}$[a]	$C_{L\alpha_{th}}$[a]	c, ft	$b/2$, ft
1	2.0	1.0	1.204	2.16	2.29	0.5	0.5
2	3.0	1.0	1.181	2.83	2.97	0.5	0.75
3	4.0	1.0	1.163	3.44	3.43	0.5	1.0
4	5.0	1.0	1.151	3.76	3.78	0.5	1.25
5	6.0	1.0	1.137	3.88	4.02	0.5	1.5
6	6.0	0.75	1.051	4.01	4.10	0.571	1.5
7	6.0	0.5	0.959	4.30	4.16	0.666	1.5
8	6.0	0.25	0.861	4.17	4.18	0.8	1.5
9	6.0	0.0	0.755	3.63	3.96	1.0	1.5

[a] Per radian.

The strength of the rolled-up tip vortex can also be related to the lift distribution over a wing. The wings considered were of two types: fixed aspect ratio with varying taper ratio, and fixed taper ratio with varying aspect ratio. Reference 5 describes a method for predicting the span-load distribution of tapered wings. This technique was used to determine the wing and section lift characteristics for all of the wings under consideration. Reference 6 presents the spanwise lift distributions for many geometries of unswept wings, several of which were considered in this investigation. The midspan values of the section lift coefficient derived from this source were in general agreement with the previous results. For the effect of aspect ratio on the lift of a wing, McCormick[7] offers a first-order correction to lifting-line theory for large-aspect-ratio wings or to slender wing theory for low-aspect-ratio wings.

Several of the characteristics just discussed are presented in Table 1 for all of the wings considered. Using the results of the previous discussions, the circulation was determined for wings 1–9 at a freestream velocity of 75 mph. These predicted values of the circulation as a function of aspect ratio and taper ratio are shown in Fig. 3.

Boundary-Layer Analysis

The flow of fluid in the neighborhood of a finite-span lifting surface exemplifies a very complex three-dimensional flow. Chordwise and spanwise pressure gradients affect the flow differently at various distances normal to the surface. The flow in the chordwise direction is retarded within the boundary layer, and will be deflected in the spanwise direction to a greater extent than the flow outside the boundary layer. The difference between the spanwise component of velocity in the boundary layer and that in the external flow is the crossflow. In general the crossflow is large in the region of the wing tip. Cooke and Hall[8] discuss several methods for computing the three-dimensional boundary layer, all of which have two common criteria: the external streamlines of the flow must be specified, and the crossflow must be small. Both of these requirements are difficult to satisfy near the wing tip. These facts preclude any theoretical three-dimensional boundary-layer analysis. However, one observation applicable to this study has been made. McCormick[9] concluded that the thickness of the tip vortex core is governed by the thickness of the boundary layer at the trailing edge on the lower surface of the wing, and notes that thickening the vortex core reduces the tangential velocity of the rolled-up vortex.

An investigation into the two-dimensional boundary layer on the same wings, using techniques of Pohlhausen,[10] Launder,[11] and Granville,[12] revealed that the boundary layer on the lower surface of the wing remains laminar up to a point of separation near the trailing edge. Transition did not occur. This condition was verified by the flow-visualization study. The presence of separated flow on the lower surface prevented the derivation of any approximate relationship between the thickness of the boundary layer and the width of the vortex.

Vorticity Meter Calibration

Previous vortex analyses[1,2] have considered calibration of the vorticity meter to correct inaccuracies in experimental data due to finite probe size and friction and windage losses. In those analyses, the nondimensional axial-velocity ratio V_x/V was assumed to be constant and equal to one throughout the flowfield. More recently Logan[3] found that within the vortex there is both acceleration and retardation of the axial velocity. Thus, if the axial-velocity ratio is considered constant at any one point in the field, but not necessarily equal to one, then the calibration factor becomes

$$\eta = \frac{1}{1 + k/\bar{V}V} \tag{5}$$

where η is the ratio of the rotational velocity of the vanes to that of the fluid, $\bar{V}$ is the axial velocity ratio, and k is a constant shown in the previous papers to have the value 18.8. This approach assumes that the axial velocity experienced by each of the vanes can be represented by that which occurs at the centerline of the probe.

Experimental Investigation and Discussion

The experimental investigation was conducted in the subsonic wind tunnel of the Department of Aerospace Engineering at the Pennsylvania State University. The test section was operated at approximately atmospheric pressure and at a velocity of 75 mph.

The wings tested were basically five in number, untwisted and unswept, constant NACA 0012 airfoil sections, of varying aspect ratio and taper ratio. Four of the wings were a constant aspect ratio of 6 varying in taper ratio from 0.0 to 0.75. A fifth wing had a fixed taper ratio of 1 and was so constructed that the aspect ratio could be varied between 1 and 6 by the addition or elimination of sections. Table 1 lists each wing configuration and some of the experimentally and theoretically determined characteristics of each. Lift data as a function of angle of attack were obtained using a six-component strain-gage balance.

In consequence of McCormick's conclusion[9] regarding the boundary layer on the lower surface and its effect on the tip vortex, a series of tests were conducted in which transition strips were located at several chordwise locations on the lower surface to determine the effect of a turbulent boundary layer. The locations were chosen arbitrarily as the section midchord and a point near the leading edge behind the stagnation point. The strips were of sufficient size and density to insure fully developed turbulent flow at the strip, as verified by flow-visualization studies, but not so large that a measurable increase in drag would result. No significant change in wing lift was noted. The following sections describe the primary experimental apparatus, its function and limitations.

Vorticity Meter

The vorticity meter, or vortex probe, used for this investigation was developed by May[13] in 1964. It consists of four unpitched steel vanes mounted on an aluminum spinner (Fig. 4). A hole drilled through the shaft allows the passage of light from a miniature light bulb to a photovoltaic cell as the shaft rotates. These pulses are counted electronically as cycles per second. The purpose of the probe was to measure the vorticity in the rolled-up vortex at a selected downstream location as the aspect ratio, taper ratio, and boundary conditions on the lower surface of the wing were altered upstream. Data were taken at small spanwise intervals as the probe moved in a plane through the center of the vortex. Figure 5 shows schematically the experimental arrangement. Tangler[2] provides a more complete description of the vorticity meter, the accompanying support and drive mechanism, and their operation. The resulting vorticity data were reduced using the data-reduction program of Tangler.

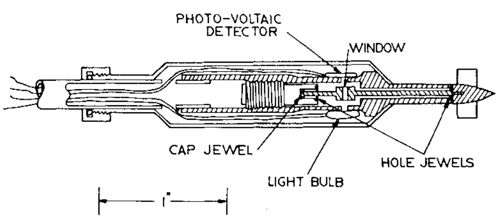

Fig. 4 Schematic of the vorticity meter.

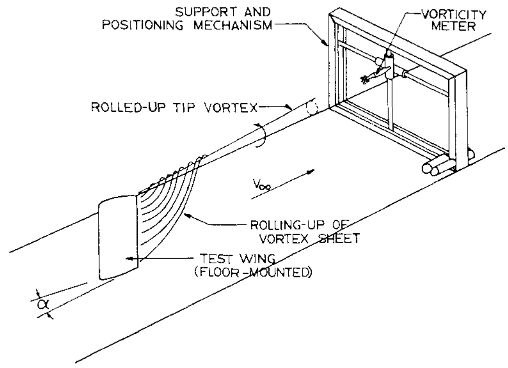

Fig. 5 Experimental arrangement.

For all tests the vorticity probe was positioned 40 in. downstream of the wing quarter-chord. The primary criterion was that the probe be sufficiently far downstream to insure complete rollup of the vortex sheet prior to measurement.

Velocity Probe

The apparatus necessary for another phase of the experimental investigation was a five-holed static-pressure probe attached to a positioning structure having four degrees of freedom. Yaw and pitch control were achieved by two orthogonal arcs, whereas linear positioning was obtained using the traverse mechanism of Tangler.[2] A detailed description of the equipment and its operation is given by Logan.[3] The purpose of this investigation was to determine the axial and tangential velocities within the rolled-up vortex so that the accuracy of the data obtained with the vortex probe could be established and the calibration evaluated. The probe was positioned 40 in. downstream of the wing quarter-chord.

Flow-Visualization Study

The purpose of the flow-visualization tests was to obtain qualitative information concerning flow deflection, cross flow, and boundary-layer characteristics, all as a function of wing inclination and geometry and the position of transition on the lower surface of the wing. It was assumed that any significant difference in flow pattern could be associated with a corresponding trend in the vortex strength and geometry.

The technique used to observe the flow in the boundary layer was as discussed by Loving and Katzoff.[14] It consisted of coating a surface with a film of fluorescent oil and observing where the thickness was affected by the shearing action of the boundary layer. The resultant film can be detected by use of ultraviolet light.

Results and Discussion

The data presented in the following discussion were obtained by the vortex probe. The symmetry of the experimentally measured vorticity distribution led to the conclusion that the vortex sheet was completely rolled up in all instances. The axial and tangential velocity distributions measured in the rolled-up vortex with the velocity probe were used to

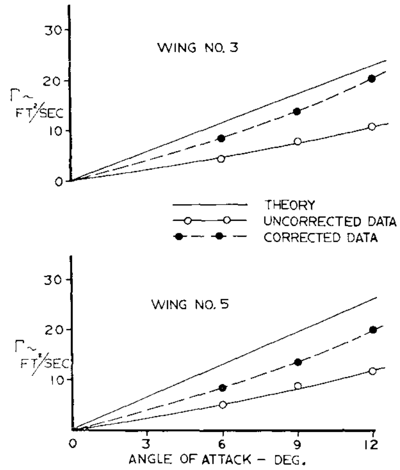

Fig. 6 Effect of calibration on experimental data.

evaluate the accuracy of the data obtained with the vorticity meter, and to establish the required correction for calibration of the vorticity probe. The effect of calibration on the values of the measured circulation is shown in Fig. 6 for two wing configurations.

Figure 7 shows typical experimental results, with and without corrections for calibration, and indicates that the circulation in the vortex increases with increasing aspect ratio and taper ratio for all cases. One apparent result is that the uncorrected experimental values are at most 50% of those predicted. This is consistent with the results of several other studies[2,15] and is as yet unexplained. The maximum value of the tangential velocity in the vortex (Fig. 8) was found to follow trends similar to the circulation. The data indicated that the maximum value of the tangential velocity in any of the cases considered was about 40% of the freestream velocity. Although no experimental data are shown for cases with transition on the lower surface, it was found that the values of the circulation and maximum tangential velocity were maximum for the smooth lower surface, decreased as transition moved forward to the midchord, and then increased as transition approached the leading edge. This variation is shown in Ref. 16, but remains unexplained. It is worth noting that the wings with transition at the midchord of the lower surface exhibited tangential velocities that generally were 10–30% lower than those corresponding to wings with no transition on the lower surface.

From the data, it was apparent that the width of the vortex increased as the line of transition moved toward the leading edge of the wing. This is in agreement with McCormick's

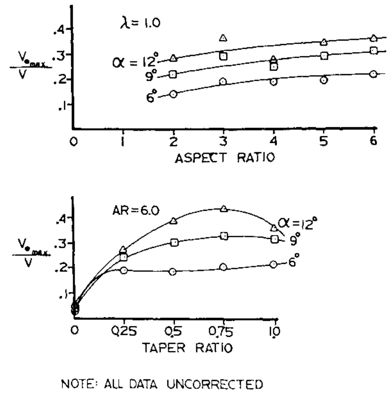

Fig. 8 Experimental variation of the maximum tangential velocity with aspect ratio and taper ratio.

conclusion. It should be noted (Fig. 9) that the width of the vortex was almost independent of aspect ratio, and is unaltered by calibration corrections.

From the foregoing figures, the strength and size of the rolled-up tip vortex can be predicted for any wing of comparable geometry and Reynolds number. Prediction of these characteristics for full-scale aircraft has presented several problems. However, Sherrieb[15] has attempted to correlate these model data with the trailing vortex systems of full-scale aircraft.

From the flow-visualization study no significant difference in the deflection of the streamlines was apparent between wings of varying aspect ratio and taper ratio for similar boundary-layer conditions. Figure 10 shows wing 2 with transition at the midchord on the lower surface. One of the most evident details is the extreme deflection of the streamlines in the boundary layer at the wing tip. Note the difference in the spanwise deflection of the flow ahead of and behind the transition strip.

Conclusions

The purpose of this investigation was to determine the effect the wing geometry and the boundary layer on the lower surface of the wing have on the size and strength of the rolled-up tip vortex. The following conclusions can be drawn.

1) The size and strength of the rolled-up tip vortex of any wing can be predicted if the geometry of the wing and the Reynolds number are comparable to those of the wings tested.

2) The circulation and the maximum tangential velocity of the rolled-up vortex increase with increasing aspect ratio and taper ratio.

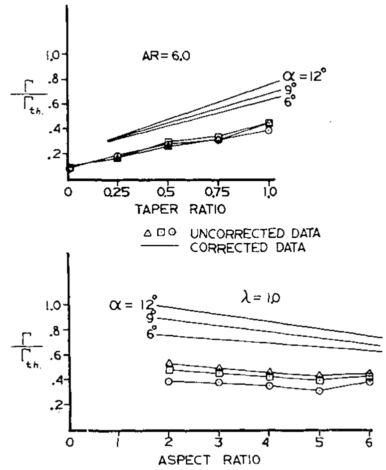

Fig. 7 Experimental variation of the circulation with aspect ratio and taper ratio.

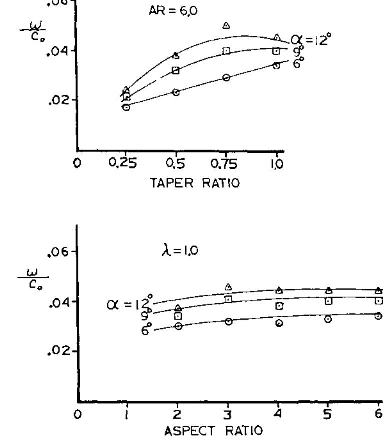

Fig. 9 Experimental variation of the vortex width with aspect ratio and taper ratio.

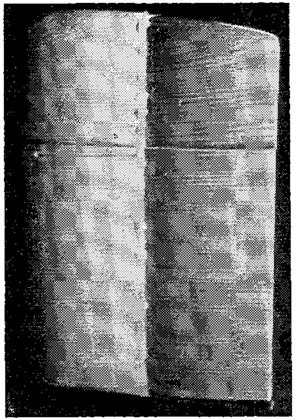

Fig. 10 Wing no. 2 with transition at the mid-chord; $\alpha = 12°$.

3) The width of the vortex is nearly independent of aspect ratio. It also increases for increasing taper ratio and increasing proportions of turbulent flow on the lower surface of the wing.

4) The results regarding the lower-surface boundary-layer effects on the tip vortex are generally inconclusive.

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